A New Monte Carlo Circuit Simulation Paradigm With Specific Results for Resistive Networks

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***Abstract*—In this paper, we formulate a new type of Monte Carlo problem for circuits. Specific results are given for the class of resistive networks and open research problems are indicated for more general cases. Given lower and upper bounds on the value of each resistor but no probability distribution, we consider the problem of estimating appropriate probabilistic measures of performance. In view of the fact that no *a priori* probability distributions for the uncertain resistors are assumed, a certain type "distributional robustness" is sought. To this end, a new paradigm from the robustness literature is particularized to these circuits. Some of the performance bounds obtained via this new approach differ considerably from those which result from a more conventional Monte Carlo simulation.**

***Index Terms*—Circuit simulation, control theory, ladder networks, Monte Carlo methods, optimization, resistive circuits, robustness.**

I. Introduction and Formulation

A TYPICAL Monte Carlo simulation for a circuit begins with a specified vector of uncertain parameters

$$\theta \doteq (\theta_1, \theta_2, \ldots, \theta_n)$$

a smooth performance measure $\phi(\theta)$ and a specified probability density function $f(\theta)$ for θ; e.g., see [13] for the general context and [15] for specifics to circuits. Accordingly, in a classical simulation, N samples $\theta^1, \theta^2, \ldots, \theta^N$ of θ are generated using the probability density function $f(\theta)$ and subsequently, various performance estimates are obtained. For example, the relative frequency estimate

$$\hat{\phi}_N \doteq \frac{1}{N}\sum_{i=1}^{N}\phi(\theta^i)$$

can be used to estimate the mean performance

$$\mathcal{E}\phi \doteq \int_\theta \phi(\theta)f(\theta)\,d\theta$$

and an estimate

$$\hat{V}_N \doteq \frac{1}{N}\sum_{i=1}^{N}(\phi(\theta^i) - \hat{\phi}_N)^2$$

for the variance

$$\mathcal{E}(\phi - \mathcal{E}\phi)^2 \doteq \int_\theta (\phi - \mathcal{E}\phi)^2 f(\theta) d\theta$$

is obtained in a similar manner. Subject to various conditions on how large N should be, the estimates above can be "certified" in terms of their reliability.

A. Fundamental Issues in This Paper

The classical Monte Carlo simulation described above involves assuming an *a priori* probability density functions $f(\theta)$ for θ. In contrast, this paper addresses the case when there is no such *a priori* information available. Moreover, unlike many classical Monte Carlo approaches to problems without *a priori* probability distributions, in this paper, we do not simply impose some "reasonable" distribution, such as normal or uniform, on θ. Our contention, consistent with other caveats found in the literature, is that the imposition of such an ad hoc probability distribution can lead to an unduly optimistic assessment of performance; e.g., see [12] and [15].

In this paper, we provide a new paradigm for probabilistic assessment which we believe leads to more realistic estimates of performance in the absence of such *a priori* information. Our method is seen to be *distributionally robust* in the following sense: With performance uncertainty as described above, the performance estimate which we define is guaranteed for all probability distributions f in a given class $\mathcal{F}$. The definition of $\mathcal{F}$, initially provided in [3], is felt to be particularly apropos to circuits for which components are described by tolerances as in [6]. To this end, the definition of $\mathcal{F}$ is based on the assumption that random circuit parameters θ_i vary independently with known bounds $\theta_i^- \leq \theta \leq \theta_i^+$, the intuitive notion that positive and negative deviations from the nominal $\theta_i = \bar{\theta}_i$ manufacturing value are equally likely, and that the larger the deviation in θ_i from $\bar{\theta}_i$, the less likely it is to occur.

Within this setting, we take Θ to be the hypercube in $\mathbb{R}^n$ associated with these bounds and it is assumed that any $f \in \mathcal{F}$ which is an *admissible* density function is supported in Θ. It is noted that this definition of $\mathcal{F}$ is non-parametric. In fact, the given data in this problem formulation is precisely the same as that used

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in robust systems analysis: Interval bounds without a statistical description of the underlying probability distribution. With the set-up above, unlike classical Monte Carlo approaches, we do not conduct a simulation with an assumed probability distribution $f \in \mathcal{F}$ for θ. Instead, we develop a simulation theory which is robust with respect to $f \in \mathcal{F}$. To this end, with θ^f denoting the random vector with probability density function $f \in \mathcal{F}$ and low ϕ values denoting "good performance," we consider a pair of expected performance problems: For the mean, we seek

$$\max_{f \in \mathcal{F}} \mathcal{E}(\phi(\theta^f)) \doteq \max_{f \in \mathcal{F}} \int_\theta \phi(\theta) f(\theta)\, d\theta$$

and for the variance, we seek

$$\max_{f \in \mathcal{F}} \mathcal{E}(\phi(\theta^f) - \mathcal{E}(\phi(\theta^f)))^2 \doteq \max_{f \in \mathcal{F}} \int_\theta (\phi(\theta) - \mathcal{E}(\phi(\theta^f)))^2 f(\theta)\, d\theta.$$

We call the optimal values above *distributionally robust* estimates and note that the solution to these problems leads to a so-called *a posteriori* Monte Carlo simulation. For example, if one wishes to estimate the sharpest possible distributionally robust upper bound on the expected performance function, one uses a solution

$$f^* \in \arg\max_{f \in \mathcal{F}} \mathcal{E}(\phi(\theta^f))$$

to generate the samples θ^i entering into the estimate $\hat{\phi}_N$. We draw attention to the fact that f^* might differ significantly from the distribution one might select as being "reasonable" in the absence of *a priori* information. By way of illustration, if one considers a simple voltage divider relationship $\phi(R_1, R_2) = R_2/(R_1 + R_2)$ to define performance, it is clear that the average value of ϕ depends critically on what probability distributions are imposed on the two resistors R_1 and R_2. In the context of this example, the objective in this paper is to *assign* probability distributions f_1 for R_1 and f_2 for R_2 which lead to estimated performance that is "robust" in the sense described above; see numerical examples in Section V.

B. *Specific Results*

The success of the paradigm above is predicted upon the ability to obtain a density $f^* \in \mathcal{F}$ achieving distributional robustness. In the remainder of this paper, we describe large classes of resistive networks for which f^* is readily obtained. This motivates further research for general classes of circuit problems beyond those considered here. In the sequel, we provide a rather detailed analysis of gain computation for resistive networks and we provide a precise characterization of f^* for both the mean and variance measures above. We conclude this paper by pointing out some open problems at the level of resistive networks and suggest fruitful directions of research for more general networks.

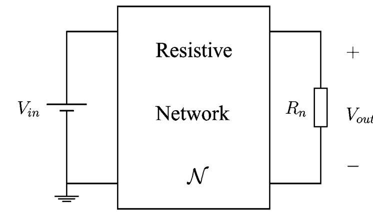

Fig. 1. Network configuration.

II. Resistive Networks

A. *Formulation*

We consider a planar resistive network $\mathcal{N}$ consisting of an input voltage source V_{in}, an output voltage V_{out} across a designated resistor $R_{\text{out}} = R_n$ and uncertain resistors

$$R \doteq (R_1, R_2, \ldots, R_n)$$

as depicted in Fig. 1. To make an identification with general formulation in Section I, we take $\theta = R$. While working with only a single independent voltage source and a planar network for simplicity of exposition, it is noted the results in this paper are readily modified to accommodate a variety of other situations.

B. *Uncertain Resistors*

To describe the uncertainty, for each resistor, we consider *nominal manufacturing value* $\bar{R}_i > 0$ for R_i and interval uncertainty bounds

$$R_i \in \mathcal{R}_i \doteq [\bar{R}_i - r_i, \bar{R}_i + r_i]$$

with $r_i < \bar{R}_i$ to guarantee positive resistance. This leads to the hypercube

$$\mathcal{R} \doteq \mathcal{R}_1 \times \mathcal{R}_2 \times \cdots \times \mathcal{R}_n.$$

C. *Admissible Probability Densities*

Consistent with Section I, it is assumed the R_i are independent random variables supported in $\mathcal{R}_i$ with an unknown probability density function $f_i(R_i)$ which is symmetric about its mean $\bar{R}_i$. It is also assumed that positive and negative deviations away from $\bar{R}_i$ are equally likely and that $f_i(R_i)$ is non-increasing in $|R_i - \bar{R}_i|$; i.e., large deviations from the mean $\bar{R}_i$ are no more likely than small deviations. It is noted that this formulation allows each R_i to have a different probability density function. We write $f \in \mathcal{F}$ to denote an *admissible joint density function* $f(R)$ over $\mathcal{R}$. Given any $f \in \mathcal{F}$, the resulting random vector of resistors is denoted as R^f. Two important special cases of interest are obtained when $f = u_i$ is the uniform distribution centered at $\bar{R}_i$ and $f = \delta_i$ is the Dirac impulse distribution centered at $\bar{R}_i$.

D. Distributional Robustness

Within the new Monte Carlo framework of this paper, the objective is to compute what we call "distributionally robust limits" for the expected gain g and its variance. More specifically, for the system of Fig. 1, we first focus attention on the uncertain gain

$$g(R) \doteq \frac{V_{\rm out}}{V_{\rm in}}.$$

Now, given any $f \in \mathcal{F}$, the associated multi-dimensional integral for the *expected gain* is given by

$$\mathcal{E}(g(R^f)) = \int_{\mathcal{R}} g(R)f(R)\,dR.$$

Similarly, for the case of variance, we obtain

$$\mathcal{V}(g(R^f)) = \int_{\mathcal{R}} (g(R) - \mathcal{E}(g(R^f)))^2 f(R)\,dR.$$

Within this framework, we seek to compute the expected gain limits

$$g^- \doteq \min_{f\in\mathcal{F}} \mathcal{E}(g(R^f)), \quad g^+ \doteq \max_{f\in\mathcal{F}} \mathcal{E}(g(R^f))$$

and maximal variance

$$v^+ \doteq \max_{f\in\mathcal{F}} \mathcal{V}(g(R^f)).$$

We do not consider minimization of the variance because it is trivially obtained by setting f to Dirac impulse distribution.

E. Monte Carlo Implication

A solution $f^* \in \mathcal{F}$ to one of the variational problems above defines the probability distribution to be used in a subsequent Monte Carlo simulation. In other words, rather than assume an *a priori* probability distribution for simulation, we obtain an *a posteriori* distribution which is the solution of one of the variational problems above.

III. Essential Resistors, Truncations, and Main Results

In order to convey the main results, two definitions are required. To this end, we define the notion of "essential" resistors and "truncated uniform distributions."

A. Essential Resistors

A resistor R_k is said to be *essential* if the following condition holds: There do not exist admissible values of the $(n-1)$ remaining resistors $R_i^* \in \mathcal{R}_i, i \neq k$, making the gain $g(R)$ independent of $R_k \in \mathcal{R}_k$. To this end, it can readily be shown that the following statements are equivalent:

- R_k is essential;
- the current through R_k and the partial derivative $\partial g/\partial R_k$ are non-zero over $\mathcal{R}$;
- the current through R_k and the partial derivative $\partial g/\partial R_k$ have one sign over $\mathcal{R}$.

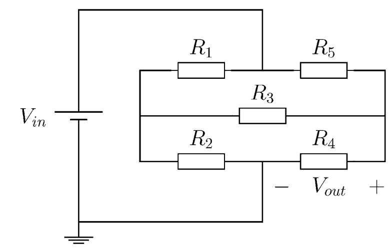

Fig. 2. Essentiality condition.

In view of this, let

$$s_k \doteq \text{sign}\left(\frac{\partial g}{\partial R_k}\right)$$

denote this invariant sign; i.e., s_k is constant over $\mathcal{R}$ having the value $s_k = -1$ or $s_k = 1$.

B. Example

To illustrate the ideas involved in the definition above, we consider the Wheatstone bridge circuit of Fig. 2. This circuit is said to be "balanced" when $(R_1)/(R_2) = (R_5)/(R_4)$. When the circuit is balanced, the voltage across the resistor R_3 is zero (no current), which makes the resistor R_3 nonessential. On the other hand, the gain of the circuit is computed to be $g(R) = N(R)/D(R)$, where

$$N(R) = R_4(R_1R_2 + R_1R_3 + R_2R_3 + R_2R_5)$$

and

$$D(R) = R_1R_2R_4 + R_1R_2R_5 + R_1R_3R_4 + R_1R_3R_5 + R_1R_4R_5 + R_2R_3R_4 + R_2R_3R_5 + R_2R_4R_5.$$

Now, to illustrate the use of the essentiality definition, we consider R_1 with associated partial derivative $\partial g/\partial R_1 = N_1(R)/D_1(R)$, where

$$N_1(R) = -R_2R_4R_5(R_2R_5 + R_3R_4 + R_3R_5 + R_4R_5)$$

and

$$D_1(R) = (R_1R_2R_4 + R_1R_2R_5 + R_1R_3R_4 + R_1R_3R_5 + R_1R_4R_5 + R_2R_3R_4 + R_2R_3R_5 + R_2R_4R_5)^2.$$

Now, from the expression above, it is immediate that this partial derivative has one sign, $s_1 = -1$. Hence, we conclude that R_1 is essential.

To demonstrate how nonessentiality arises, we consider the resistor R_3 with associated partial derivative $\partial g/\partial R_3 = N_3(R)/D_3(R)$, where

$$N_3(R) = R_4R_5(R_1 + R_2)(R_1R_4 - R_2R_5)$$

and

$$D_3(R) = (R_1R_2R_4 + R_1R_2R_5 + R_1R_3R_4 + R_1R_3R_5 + R_1R_4R_5 + R_2R_3R_4 + R_2R_3R_5 + R_2R_4R_5)^2.$$

Now, in view of the term $(R_1R_4 - R_2R_5)$ above, the range of parameter variation becomes critical in the determination of essentiality. For example, with parameter uncertainty bounds $\mathcal{R}_1 = \mathcal{R}_4 = [90, 100]$ and $\mathcal{R}_2 = \mathcal{R}_5 = [40, 50]$, essentiality of

R_3 is guaranteed (the circuit cannot be balanced). On the other hand, with $\mathcal{R}_1 = \mathcal{R}_2 = \mathcal{R}_4 = \mathcal{R}_5 = [90, 100]$, resistor R_3 becomes nonessential.

Finally, to complete the analysis of this example, we note that calculations similar to those above lead to essentiality of R_1, R_4 and R_5, irrespective of the range of parameter variation. For the essential resistors R_1, R_2, R_4, and R_5, the corresponding signs are $s_1 = s_5 = -1$ and $s_2 = s_4 = 1$.

C. Truncated Uniform Distributions

As described in [3] and [10], there are large classes of distributional robustness problems having the property that extremization with respect to $f \in \mathcal{F}$ is equivalent to extremization over the class of so-called *truncated uniform distributions*. More specifically, letting

$$T \doteq [0, r_1] \times [0, r_2] \times \cdots \times [0, r_n]$$

we define R^t to be the random vector with probability density function which is uniform over the *truncated hypercube*

$$\mathcal{R}^t \doteq \mathcal{R}_1^{t_1} \times \mathcal{R}_2^{t_2} \times \cdots \times \mathcal{R}_n^{t_n}$$

where

$$\mathcal{R}_i^{t_i} \doteq [\bar{R}_i - t_i, \bar{R}_i + t_i].$$

In other words, each component R_i of R has a uniform distribution over the sub-interval $\mathcal{R}_i^t$ of $\mathcal{R}^i$. In the sequel, we make use of the fact for large classes of performance measures $\phi(R)$, it is known that

$$\begin{aligned}\max_{f \in \mathcal{F}} \phi(R^f) &= \max_{t \in T} \phi(R^t), \\ &= \max_{t \in T} \frac{1}{2^n t_1 t_2 \cdots t_n} \int_{\mathcal{R}^t} \phi(R)\, dR\end{aligned}$$

with the understanding that if $t_i = 0$, the corresponding integral with $(1)/(2t_1)$ multiplier is calculated using an appropriate impulse distribution or l'Hôpital's rule. This result, known as the *Truncation Principle*, also holds for the case of minimization.

D. Remark

The truncation concepts above, combined with essentiality considerations, are central to the proofs of the results given below. For the case of an essential resistor R_i, both the maximum and minimum expected gains are attained with a probability density functions f_i^* either being an impulse at $\bar{R}_i$ or uniform over $\mathcal{R}_i$. In other words, we obtain a solution which is extremal over the class of truncated uniform distributions. In view of the fact that such distributions can be viewed as extreme within the class $\mathcal{F}$, we call the results below *extremality theorems*.

For the case of the variance of the gain, a weaker result is given. Since we cannot assure that the maximum over $f \in \mathcal{F}$ is equal to maximum over $t \in T$, we allow for the possibility of a "gap" (see conclusions in Section VI) and consider the problem of finding the maximum of $\mathcal{V}(g(R^t))$ with respect to $t \in T$. Again, an extremality result is obtained in the sense that the optimal distribution turns out to be uniform.

E. Gain Extremality Theorem (See Appendix I for Proof)

Let R_i be an essential resistor in $\mathcal{N}$. For the case of maximizing $\mathcal{E}(g(R^f))$, define probability density function f^* with marginals f_i^* as follows: Set $f_i = u_i$ if $s_i = -1$ and $f_i^* = \delta_i$ if $s_i = 1$. Then

$$\max_{f \in \mathcal{F}} \mathcal{E}(g(R^f)) = \mathcal{E}(g(R^{f^*})).$$

For the case of minimizing $\mathcal{E}(g(R^f))$, define probability density function f^* with marginals f_i^* as follows: Set $f_i = \delta_i$ if $s_i = -1$ and $f_i^* = u_i$ if $s_i = 1$. Then,

$$\min_{f \in \mathcal{F}} \mathcal{E}(g(R^f)) = \mathcal{E}(g(R^{f^*})).$$

F. Variance Extremality Theorem (See Appendix III for Proof)

For $t \in T$, let $\mathcal{V}(g(R^t))$ denote the variance of the gain with truncated probability distribution $f = u^t$. Then, with $f^* = u$ being the uniform distribution, we obtain

$$\max_{t \in T} \mathcal{V}(g(R^t)) = \mathcal{V}(g(R^u)).$$

IV. Essentiality Considerations

We emphasize that the use of Theorem III-E is predicted upon essentiality. Accordingly, in this section, we provide some results, based on the linear equation analysis in [8], which enables us to determine which resistors R_i in a network are essential and, when so, what corresponding sign s_i should be used for the purpose of Monte Carlo simulation. To this end, the following lemma, plays an important role.

A. Lemma (See Appendix III for Proof)

The partial derivative of the gain $g(R)$ with respect to resistor R_k admits a factorization of the form

$$\frac{\partial g}{\partial R_k} = \frac{g_1(R) g_2(R)}{h(R)}.$$

with $g_1(R)$ and $g_2(R)$ being multilinear functions and $h(R)$ being a positive function.

B. Remark

In order to use the result above to test for essentiality, we evaluate $g_1(R)$ and $g_2(R)$ at the vertices of $\mathcal{R}$. Now using the well-known result that a multilinear function on a hypercube is both maximized and minimized at its vertices, (for example, see [1]), we obtain the following: A necessary and sufficient condition for essentiality is that all of the vertex evaluations of

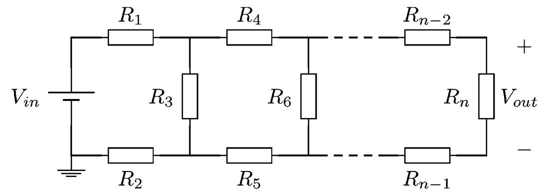

Fig. 3. Resistive ladder network.

$g_1 g_2$ are non-vanishing and of the same sign. This being the case, we obtain

$$s_k = \operatorname{sign}(g_1 g_2) = \operatorname{sign}(g_1)\operatorname{sign}(g_2)$$

for the probability density function assignment in Theorem III-E. Note that this sign condition is often verifiable by inspection. To illustrate, in Example III-B, when considering R_3, since $g_1(R) = R_4 R_5 (R_1 + R_2)$, it is immediate by inspection that $\operatorname{sign}(g_1) = 1$. For further details illustrating this type of analysis in the more general context of linear equations, see [8].

V. Classes of Networks

Using the ideas in the preceding two sections, it is often possible to develop a prescription for distributionally robust Monte Carlo simulation which applies to an entire class of networks. To illustrate, we now consider the case of ladder networks; see [9] for preliminary version of the results in this section.

A. Theorem (See Appendix IV for Proof)

Consider the multi-stage ladder network of Fig. 3. For the case of maximizing $\mathcal{E}(g(R^f))$, define probability density function f^* with marginals f_i^* as follows: Set $f_i^* = \delta_i$, the Dirac delta distribution at $\bar{R}_i$, for the inter-stage resistors $(R_3, R_6, \ldots, R_n)$ and $f_i^* = u_i$, the uniform distribution centered at $\bar{R}_i$, for the remaining resistors. Then,

$$\mathcal{E}(g(R^{f^*})) = \max_{f \in \mathcal{F}} \mathcal{E}(g(R^f)) = g^+.$$

For the case of minimizing $\mathcal{E}(g(R^f))$, define probability density function f^* with marginals f_i^* as follows: Set $f_i^* = u_i$ for the inter-stage resistors $(R_3, R_6, \ldots, R_n)$ and $f_i^* = \delta_i$ for the remaining resistors. Then

$$\mathcal{E}(g(R^{f^*})) = \min_{f \in \mathcal{F}} \mathcal{E}(g(R^f)) = g^-.$$

B. Remarks

The results above, for the ladder network, may appear counterintuitive in the sense that one should resist the temptation to sample the range of variation for the interstage resistors; i.e., while drawing samples uniformly distributed for the non-interstage resistors, one should hold the interstage resistors fixed at their nominal values. In this regard, our claim is that a more classical Monte Carlo simulation, involving uniform sampling of all resistors, provides an unduly optimistic result from a distributional robustness point of view. By way of future research, it would be of interest to analyze other classes of networks to determine the extent to which results along the lines of Theorem V-A can be given. For example, one can identify various classes of lattice networks or filters leading to essentiality of all resistors. For such cases, one immediately obtains a prescription for distributionally robust Monte Carlo simulation.

To conclude the discussion of ladder networks, it is interesting to compare the worst-case gain

$$g_{\max} \doteq \max_{R \in \mathcal{R}} g(R)$$

with the distributionally robust gain. Indeed, for the case of maximizing $g(R)$, it is readily verified that g^* is attained if we define R^* with components $R_i^* = \bar{R}_i + r_i$ for the inter-stage resistors $(R_3, R_6, \ldots, R_n)$ and $R_i^* = \bar{R}_i - r_i$ for the remaining resistors. As seen in the example below, the difference between the distributionally robust expected gain and the worst-case gain can be quite large.

C. Numerical Example

The ideas above are now illustrated for a three stage ladder network with nominal values $\bar{R}_1 = \bar{R}_4 = \bar{R}_5 = \bar{R}_7 = \bar{R}_8 = 1, \bar{R}_2 = 2, \bar{R}_3 = 3, \bar{R}_6 = 5$, and $\bar{R}_9 = 7$. To illustrate how significantly different a classical Monte Carlo simulation can be versus our method, we consider specially constructed uncertainty bounds $\mathcal{R}_1 = \mathcal{R}_4 = \mathcal{R}_5 = \mathcal{R}_7 = \mathcal{R}_8 = [0.9, 1.1], \mathcal{R}_2 = [1.8, 2.2], \mathcal{R}_3 = [0.6, 5.4], \mathcal{R}_6 = [1, 9]$, and $\mathcal{R}_9 = [1.4, 12.6]$. As prescribed by Theorem V-A, we carry out a Monte Carlo simulation using an impulsive distribution for R_3, R_6 and R_9 and a uniform distribution for the remaining R_i. With 100 000 samples, we obtained the estimate $\mathcal{E}(g(R^{f^*})) \approx 0.1863$.

Next, for comparison purposes, a classical Monte Carlo simulation using the uniform distribution for all resistors was carried out. This time, an estimate $\mathcal{E}(g(R^u)) \approx 0.1555$, was obtained. In conclusion, it is apparent that the classical expected gain is less than the distributionally robust expected gain by about 17%. We also note that in both cases the estimated expectation rapidly converges; e.g., see Fig. 4 for a convergence plot corresponding to our distributionally robust simulation. Using Theorem III-F, the maximum of the variance computed to be $v^+ = 0.0038$. It is noted that the distribution maximizing the expectation of the gain is different than that maximizing the variance of the gain.

For comparison purposes, it is also noted that the maximum gain is obtained by setting $R_1 = R_4 = R_5 = R_7 = R_8 = 0.9, R_2 = 1.8, R_3 = 5.4, R_6 = 9$, and $R_9 = 12.6$. This leads to $g_{\max} \approx 0.3535$. This worst-case gain differs significantly from the distributionally robust expected gain; i.e., notice that $g_{\max}/g^+ \approx 1.8975$.

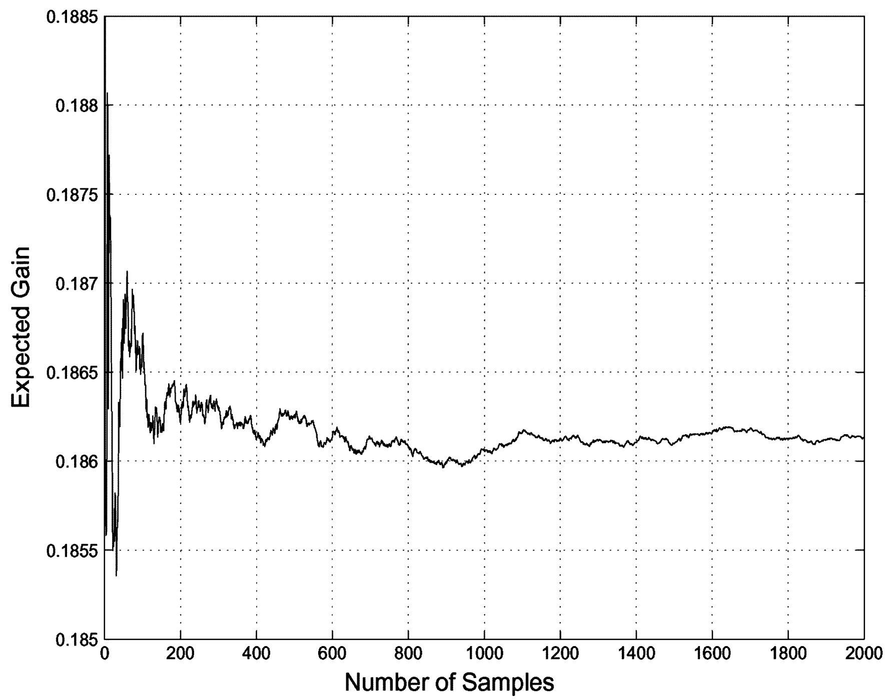

Fig. 4. Convergence of expectation for example of Section V-C.

VI. Conclusion and Further Research

To conclude this paper, we mention a number of directions for further research. First, it would be of interest to consider more general versions of the resistor problem involving other circuit elements such as capacitors and inductors. In this regard, it should first be noted that the problem formulation requires slight modification to account for the fact that the gain at any fixed frequency $\omega \geq 0$ is a complex number. To this end, the first technical point to note is that working with either the magnitude of the gain or its square, destroys the multilinear structure of the performance function, which is central to the proofs given here. To date, in this more general setting, results have only been obtained for very special cases such as first order filters with just two uncertain elements; e.g., see [16].

As demonstrated in this paper, when the resistors R_i satisfy the essentiality requirement, either the uniform or Dirac impulse distribution is used in the associated Monte Carlo simulation. By way of future research, it would be important to study this "probability distribution assignment" problem in the context of systems with nonessential elements. To this end, one starting point for this line of research is the so-called Truncation Principle described in Section III-D; e.g., see [10] for further details. For such cases, a distributionally robust solution is characterized by the so-called *truncation vector* $t \in T$ which defines a required interval of uniform sampling. To illustrate, for a nonessential resistor R_i, the question arises whether the Truncation Principle might dictate uniform sampling on a strict non-trivial subinterval of R_i. It would be of interest to determine if a counterexample exists to the conjecture that the optimal assignment for all resistors is either uniform or Dirac. By way of preliminary investigation, we revisit the circuit of Fig. 2. Let $\mathcal{R}_1 = [0,2], \mathcal{R}_2 = \mathcal{R}_5 = 1$, and $\mathcal{R}_3 = [0,4]$. It was shown in Example III-B that the only nonessential resistor is R_3. Now, suppose that we are interested in the minimum expected gain. It can be shown that $f_3^* = \delta_3$ when $R_4 = 1.4$ and $f_3^* = u_3$ when $R_4 = 1.6$. However, when $R_4 = 1.47$, using the extreme distributions, we computed $g^- \approx 0.58829$ while using a truncated uniform distribution with $t_3 = 1.6$, we obtained $g^- \approx 0.58817$. With such small difference, it is not clear whether this is an example for which extremality does not apply or a result of round-off. We note that various attempts to make such difference more dramatic were not successful.

The third open research problem which we mention relates to the so-called "gap" associated with Theorem III-F characterizing the distributionally robust variance. It would be of interest to explore the conjecture that the hypotheses of Theorem III-F guarantee satisfaction of the equality

$$\max_{f\in\mathcal{F}} \mathcal{V}(g(R^f)) = \max_{t\in\mathcal{T}} \mathcal{V}(g(R^t)).$$

In other words, the result for variance is similar to the result for gain in that no gap exists between maximization over $\mathcal{F}$ and the subclass of truncated uniform distributions.

Appendix I
Proof of Theorem III-E

The proof of the theorem is facilitated with two preliminary lemmas. The first lemma describes the multilinear structure associated with the gain of a resistive network and the second lemma provides a fundamental inequality involving a number of real scalars.

A. Lemma

For the ith resistor, defining variables $x \doteq R_i, y \doteq (R_1, R_2, \ldots, R_{i-1}, R_{i+1}, \ldots, R_n)$, the gain $g(x,y)$ can be expressed in the linear fractional form as

$$g(x,y) = \frac{A(y)x + B(y)}{C(y)x + 1}$$

with $C(y) > 0$ for all admissible y.

Proof: It is first noted that the mesh matrix M depends on x in a rank-one manner; i.e., one can write

$$M = N + xuu^T$$

with $N = N(y)$ being a square matrix depending on the remaining resistors and u being a vector obtained as follows: If x belongs to the ith loop only, then u is a vector with the ith entry equals 1 and the other entries are zero. On the other hand, if x is a shared resistor between loops i and j, then u has 1 and -1 in its ith and jth entries, respectively, and the remaining entries are zero.

Now, with $V = [1\,0\,0 \ldots\, 0]^T$ representing the source voltage, the gain takes the form

$$g(x,y) = R_{\text{out}} \eta^T M^{-1} V$$

where the vector η depends on the number of loops within which x appears. If the output resistor belongs to the ith loop only, then η is a vector with the ith entry equals 1 and the remaining entries equal zero. On the other hand, if the output resistor is shared between the ith and jth loops, then η is a vector with the ith and jth entries equal 1 and -1 respectively, and the remaining entries are zero.

The proof now proceeds by considering two cases.

Case 1: Assume $x \neq R_{\text{out}}$. Using Woodbury's identity, for example see [14], we obtain

$$M^{-1} = N^{-1} - \frac{xN^{-1}uu^T N^{-1}}{1 + xu^T N^{-1} u}$$

and $g(x,y) = f(x,y)/h(x,y)$, where

$$\begin{aligned} f(x,y) &= xR_{\text{out}}(u^T N^{-1} u \eta^T N^{-1} V - \eta^T N^{-1} uu^T N^{-1} V) \\ &\quad + R_{\text{out}} \eta^T N^{-1} V \end{aligned}$$

and

$$h(x,y) = 1 + xu^T N^{-1} u.$$

Thus

$$\begin{aligned} A(y) &= R_{\text{out}}(u^T N^{-1} u \eta^T N^{-1} V - \eta^T N^{-1} uu^T N^{-1} V) \\ B(y) &= R_{\text{out}} \eta^T N^{-1} V \\ C(y) &= u^T N^{-1} u. \end{aligned}$$

Since N is positive-definite [5], it follows that $C(y) > 0$ as required.

Case 2: Assume $x = R_{\text{out}}$. Proceeding as in Case 1, we first obtain $g(x,y) = f(x,y)/h(x,y)$, where

$$\begin{aligned} f(x,y) = x^2(u^T N^{-1} u \eta^T N^{-1} V - \eta^T N^{-1} uu^T N^{-1} V) \\ + x\eta^T N^{-1} V \end{aligned}$$

and $h(x,y)$ as in Case 1.

However, since $\eta = u$ in this case, the quadratic term can be cancelled above, leading to the simpler formula

$$g(x,y) = \frac{x\eta^T N^{-1} V}{1 + xu^T N^{-1} u}.$$

Hence, in this case, the lemma is proven with

$$\begin{aligned} A &= \eta^T N^{-1} V \\ B &= 0 \\ C &= u^T N^{-1} u. \end{aligned}$$

B. Lemma

Given positive constants $c, d, t > 0$ and $d - ct > 0$, the inequality

$$\frac{2cd}{t(d-ct)(d+ct)} - \frac{1}{t^2} \log\left(\frac{d+ct}{d-ct}\right) > 0$$

holds.

Proof: Letting

$$z \doteq \frac{d+ct}{d-ct}$$

we note that $z \geq 1$ is implied since $d - ct > 0$. Hence, the requirement of the lemma is equivalent to

$$\frac{z}{2} - \frac{1}{2z} - \log z > 0.$$

Now, considering the function

$$f(z) \doteq \frac{z}{2} - \frac{1}{2z} - \log z$$

a straightforward calculation yields

$$\frac{\partial f}{\partial z} = \frac{(z-1)^2}{2z^2} > 0.$$

Hence $f(z)$ is an increasing function in z. Since $f(1) = 0$, it follows $f(z) > 0$ for all $z > 1$. Thus, the requirement of the lemma holds.

C. Proof of Theorem

We prove the result given for the maximum g^+ while noting that a nearly identical proof can be used for the minimum g^-. Indeed, in view of existing results on probabilistic robustness, for example, see [3] and [4], the maximum of $\mathcal{E}(g(R^f))$ over

$f \in \mathcal{F}$ is equal to the maximum over truncated uniform distributions. That is, letting

$$T \doteq [0, r_1] \times [0, r_2] \times \cdots \times [0, r_n]$$

we have

$$g^+ = \max_{f \in \mathcal{F}} \mathcal{E}(g(R^f)) = \max_{t \in T} \mathcal{E}(g(R^t))$$

where R^t is the random vector with probability density function which is uniform over the *truncated hypercube*

$$\mathcal{R}^t \doteq \mathcal{R}_1^t \times \mathcal{R}_2^t \times \cdots \times \mathcal{R}_n^t$$

where

$$\mathcal{R}_i^t \doteq [\bar{R}_i - t_1, \bar{R}_i + t_1].$$

Hence,

$$g^+ = \max_{t \in T} \mathcal{E}(g(R^t)) = \max_{t \in T} \frac{1}{2^n t_1 t_2 \cdots t_n} \int_{\mathcal{R}^t} g(R)\, dR$$

with the understanding that if $t_i = 0$, the corresponding integral with $(1)/(2t_1)$ multiplier is calculated using an appropriate Dirac delta distribution or l'Hôpital's rule.

To complete the proof of the theorem, let $t^* \in T$ be the truncation corresponding to probability density functions f_i^* as prescribed in the theorem. That is, if $f_i^* = u_i$, then, $t_i^* = r_i$. Alternatively, if $f_i^* = \delta_i$, then, $t_i^* = 0$. In addition, let $t \in T$ denote any candidate truncation for the maximization of $\mathcal{E}(g(R^t))$. To show that t^* attains the maximum, it will be shown that we can replace components t_k of t with corresponding components t_k^* of t^*, one at a time, without increasing $\mathcal{E}(g(R^t))$. For example, with $n = 3$, such a sequential replacement corresponds to the sequence of inequalities

$$\begin{aligned}\mathcal{E}\left(g\left(R^{(t_1,t_2,t_3)}\right)\right) &\leq \mathcal{E}\left(g\left(R^{(t_1^*,t_2,t_3)}\right)\right)\\ &\leq \mathcal{E}\left(g\left(R^{(t_1^*,t_2^*,t_3)}\right)\right)\\ &\leq \mathcal{E}\left(g\left(R^{(t_1^*,t_2^*,t_3^*)}\right)\right).\end{aligned}$$

That is, by showing that

$$\mathcal{E}(g(R^t)) \leq \mathcal{E}\left(g\left(R^{(t_1,t_2,\ldots,t_{k-1},t_k^*,t_{k+1},\ldots,t_n)}\right)\right)$$

holds for arbitrary k, we can replace components one at a time until we arrive at the desired result

$$\mathcal{E}(g(R^t)) \leq \mathcal{E}(g(R^{t^*})).$$

Indeed, without loss of generality, we take $k = 1$ and $s_1 = -1$ noting that the proof to follow is virtually identical for $s_1 = 1$ and $k = 2, 3, \ldots, n$. Now, to separate out the dependence on R_1, we let $x \doteq R_1$ and $y \doteq (R_2, R_3, \ldots, R_n)$ and consider the conditional expectation

$$E(y, t_1) \doteq \frac{1}{2t_1} \int_{\bar{R}_1 - t_1}^{\bar{R}_1 + t_1} g(x, y)\, dx.$$

Claim: The inequality

$$E(y, t_1) \leq E(y, r_1)$$

holds for all admissible $y \in \mathcal{Y}$ where $\mathcal{Y}$ denotes the box of admissible resistor uncertainty for y. To establish this claim, in view of Lemma I-A, the conditional expectation under consideration is

$$\begin{aligned}E(y, t_1) &= \frac{1}{2t_1} \int_{\bar{R}_1 - t_1}^{\bar{R}_1 + t_1} \frac{A(y)x + B(y)}{C(y)x + 1}\, dx\\ &= \frac{1}{2t_1} \int_{-t_1}^{t_1} \frac{ax' + b}{cx' + d}\, dx'\\ &= \frac{a}{c} + \frac{bc - da}{2t_1 c^2} \log \frac{d + ct_1}{d - ct_1}\end{aligned}$$

where $a(y) \doteq A(y), b(y) \doteq A(y)\bar{R}_1 + B(y), c(y) \doteq C(y), d(y) \doteq C(y)\bar{R}_1 + 1$. Furthermore, we obtain partial derivative computed to be

$$\frac{\partial E}{\partial t_1} = (bc - ad)e(t_1)$$

where

$$e(t_1) \doteq \frac{d}{ct_1(d - ct_1)(d + ct_1)} - \frac{1}{2c^2 t_1^2} \log(\frac{d + ct_1}{d - ct_1}).$$

In view of Lemma I-B, the inequality $e(t_1) > 0$ holds. Now, in order to prove that $E(y, t_1)$ is maximized at $t_1 = r_1$, we observe that

$$\frac{\partial g}{\partial x} = \frac{ad - bc}{(cx + d)^2}$$

has negative sign $s_1 = -1$ for all y. Hence, it follows that $ad - bc > 0$ and

$$\frac{\partial E}{\partial t_1} > 0.$$

Therefore, $E(t_1, y)$ is maximized at $t_1 = r_1$ and the proof of the claim is now complete.

Finally, to complete the proof of the theorem, we now observe that it follows from the claim that

$$\begin{aligned}\mathcal{E}(g(R^t)) &= \frac{1}{2^{n-1} t_2 t_3 \cdots t_n} \int_{\mathcal{Y}} E(y, t_1)\, dy\\ &\leq \frac{1}{2^{n-1} t_2 t_3 \cdots t_n} \int_{\mathcal{Y}} E(y, r_1)\, dy\\ &= \mathcal{E}\left(g\left(R^{(t_1^*, t_2, \ldots, t_n)}\right)\right).\end{aligned}$$

APPENDIX II
PROOF OF THEOREM III-F

Following the same argument as in Appendix I, it suffices to show that the inequality

$$V(y,t_1) \le V(y,r_1)$$

holds for all admissible $y \in \mathcal{Y}$, where $\mathcal{Y}$ denotes the box of admissible resistor uncertainty for $y = (R_2, R_3, \ldots, R_n)$. In view of Lemma I-A, the conditional variance under consideration, written $V(t_1) = V(y,t_1)$ as a shorthand, is

$$V(t_1) = F(t_1) - E^2(t_1)$$

where a, b, c, and d depend on y

$$E(t_1) \doteq \frac{1}{2t_1}\int_{-t_1}^{t_1} \frac{ax+b}{cx+d}\,dx$$

and

$$F(t_1) \doteq \frac{1}{2t_1}\int_{-t_1}^{t_1} \left(\frac{ax+b}{cx+d}\right)^2 dx.$$

Now, the partial derivative is computed to be

$$\begin{aligned}\frac{\partial V}{\partial t_1} &= \frac{\partial F(t_1)}{\partial t_1} - 2E(t_1)\frac{\partial E(t_1)}{\partial t_1}\\ &= \frac{ad-bc}{t_1^2c^3}\left[a\log\frac{d+ct_1}{d-ct_1}\right.\\ &\quad + \frac{2acdt_1\left(2c^2t_1^2-d^2\right)-2bc^4t_1^3}{\left(c^2t_1^2-d^2\right)^2}\\ &\quad - 2\left\{at_1c + \frac{(bc-ad)}{2}\log\frac{d+ct_1}{d-ct_1}\right\}\\ &\quad \left.\times\left\{\frac{d}{(c^2t_1^2-d^2)} + \frac{1}{2ct_1}\log\frac{d+ct_1}{d-ct_1}\right\}\right]\\ &\doteq \frac{W(t_1)}{t_1^2c^3}.\end{aligned}$$

Letting

$$z \doteq \frac{d+ct_1}{d-ct_1}$$

we substitute

$$t_1 = \frac{d}{c}\frac{z-1}{z+1}$$

above, noting that $z > 1$ corresponds to $t_1 > 0$. Thus

$$\begin{aligned}&W(z)\\ &= (ad-bc)\left[a\log z + \frac{a}{8z^2}(z^2-1)(z^2-6z+1)\right.\\ &\quad - \frac{bc}{8dz^2}(z-1)^3(z+1)\\ &\quad + \frac{1}{4d}\{(z+1)(ad-bc)\log z\\ &\quad \left. -2ad(z-1)\}\left\{-1-\frac{1}{z}+\frac{2\log z}{z-1}\right\}\right].\end{aligned}$$

Noting that $W(z) \to 0$ as $z \to 1$, to complete the proof, it suffices to show that $(\partial W)/(\partial z) > 0$. Indeed

$$\begin{aligned}&\frac{\partial W}{\partial z}\\ &= (ad-bc)\left[\frac{(z-1)^2}{4dz^3}\{ad(z^2-z+1)\right.\\ &\quad - bc(z^2+z+1)\}\\ &\quad + \frac{a(z-1)^2}{2z^2} - \frac{(ad-bc)}{4d}\\ &\quad \times\left\{\frac{(z+1)^2}{z^2} + \frac{4\log^2 z}{(z-1)^2}\right.\\ &\quad \left.\left.+\frac{\log z(z+1)(z^2-6z+1)}{z^2(z-1)}\right\}\right]\\ &= \frac{(ad-bc)^2}{4d}\left[\frac{(z-1)^2(z^2+z+1)}{z^3} - \frac{(z+1)^2}{z^2}\right.\\ &\quad \left.-\frac{4\log^2 z}{(z-1)^2} - \frac{\log z(z+1)(z^2-6z+1)}{z^2(z-1)}\right]\\ &\doteq \frac{(ad-bc)^2}{4d}h(z).\end{aligned}$$

Now, it is straightforward to verify that $h(z) > 0$ for $z > 1$. Hence, $(\partial W)/(\partial z) > 0$ and the proof is complete.

Now, since $W(z)$ is an increasing function of z and $W(z) \to 0$, as $z \to 1$, it follows that $W(z) > 0$ for $z > 1$. Hence, $V(t_1)$ is an increasing function of t_1. Therefore, the variance is maximized by the uniform distribution.

APPENDIX III
PROOF OF LEMMA IV-A

Arguing as in the proof of Lemma I-A, for the case when $B = 0$, we have

$$\begin{aligned}AD - BC &= \eta^T N^{-1} V\\ &= \frac{\eta^T N_{\text{adj}} V}{\det(N)}\end{aligned}$$

where N_{adj} denotes the adjoint of N. Thus, we take $g_1(y) = \eta^T N_{\text{adj}} V$ and $g_2(y) = 1$. Noting that rank one dependency implies that $g_1(y)$ is multilinear in y as required. For the case when $B \neq 0$, we obtain

$$\begin{aligned}AD - BC &= R_{\text{out}}(u^T N^{-1} u \eta^T N^{-1} V\\ &\quad - \eta^T N^{-1} u u^T N^{-1} V - \eta^T N^{-1} V u^T N^{-1} u)\\ &= -\frac{\eta^T N_{\text{adj}} u u^T N_{\text{adj}} V}{\det(N)^2}.\end{aligned}$$

Thus, with $g_1(y) = -\eta^T N_{\text{adj}} u$ and $g_2(y) = u^T N_{\text{adj}} V$, we again obtain the desired multilinear factorization.

$$\Delta'(R_1,R_2,\ldots,R_n,V_{\rm in}) \doteq \begin{vmatrix} \sum_{i=1}^{3} R_i & -R_3 & 0 & \ldots & & 0 & V_{\rm in} \\ -R_3 & \sum_{i=3}^{6} R_i & -R_6 & 0 & \ldots & & 0 \\ 0 & -R_6 & \sum_{i=6}^{9} R_i & -R_9 & 0 & \ldots & 0 \\ \vdots & \ddots & \ddots & \ddots & & & \vdots \\ 0 & \ldots & & 0 & -R_{n-6} & \sum_{i=n-6}^{n-3} R_i & 0 \\ 0 & \ldots & & & 0 & -R_{n-3} & 0 \end{vmatrix}$$

Appendix IV
Proof of Theorem V-A

Using mesh analysis and Cramer's rule, the output voltage can be written as

$$V_{\rm out} = \frac{R_n \Delta'(R_1,R_2,\ldots,R_n,V_{\rm in})}{\Delta(R_1,R_2,\ldots,R_n)}$$

where the equation at the top of the page is true, and $\Delta(R_1,R_2,\ldots,R_n)$ is the determinant of the mesh matrix. It is now easy to obtain

$$\Delta'(R_1,R_2,\ldots,R_n,V_{\rm in}) = V_{\rm in} \prod_{i=1}^{\frac{(n-3)}{3}} R_{3i}.$$

Hence, the gain can be written as

$$g(R) = \frac{\prod_{i=1}^{\frac{n}{3}} R_{3i}}{\Delta(R_1,R_2,\ldots,R_n)}.$$

Thus, holding all resistors fixed except R_i, the gain is of the form

$$g(R) = \begin{cases} \frac{aR_i}{cR_i+d}, & i=3k, \quad k=1,2,\ldots,\frac{n}{3} \\ \frac{b}{cR_i+d}, & \text{otherwise.} \end{cases}$$

where a, b, c, and d are positive constants.

Now, in accordance with Theorem III-E, we compute the required s_i related to essentiality. Indeed, for the interstage resistors R_i with $i = 3k$, we calculate

$$\frac{\partial g}{\partial x} = \frac{ad}{(cx+d)^2} > 0.$$

Therefore, we obtain $s_i = 1$ implying as assignment of Dirac delta function distribution for such resisters. Similarly, for the remaining resistors R_i, we obtain

$$\frac{\partial g}{\partial x} = \frac{-bc}{(cx+d)^2} < 0.$$

Therefore, in this case, $s_i = -1$ implying an assignment of uniform distribution.

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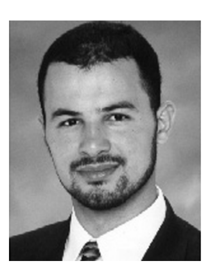

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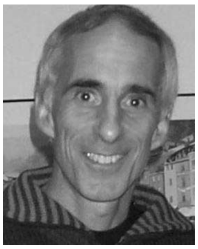

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