MIMO Ad Hoc Network Performance in the Presence of Co-channel Interference

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***Abstract*—In this paper MIMO ad hoc wireless network performance in terms of signal-to-interference-plus-noise-ratio (SINR) for two physical layers models is considered. We analyze and discuss the performance of MIMO spatial multiplexing with dominant eigenvalue transmission, when the the beamforming weights are chosen as 1) dominant eigenvector, and 2) adaptive weights that adjusted to instantaneous interference scenario. Comprehensive analytical analysis of SINR in output of a receiver node for a mentioned transmision schemes in the presence of interferer nodes are done. We show that with adaptive beamforming approach the interferer nodes signals can be significantly suppressed achieving higher network performance. The simulation result confirm our analysis.**

***Index Terms*—Ad hoc networks, array signal processing, MIMO, beamforming, channel state information, spatial multiplexing.**

I. Introduction

Wireless ad hoc networks are integral part of the next generation information infrastructure. It is basically peer-to-peer network of hosts (possibly mobile) that have neither fixed communication infrastructure nor any base stations. The nodes in these networks use distributed medium access protocols such as IEEE 802.11 to reserve local access to the wireless medium [1].

The availability of multiple antennas offers new possibilities for increasing data rate in wireless systems [2], [3]. Wireless data rate for multiple-input-multiple-output (MIMO) systems with multiple antennas at both transmitter and receiver obey the following fundamental theoretical limit: $C = M \times B \times \log(1 + SNR)$, where $M = \min\{M_t, M_r\}$, and M_t, M_r are a number of antennas at transmitter and receiver, respectively, providing an improvement of the rate by the factor M. Furthermore, MIMO technology takes advantage in propagation environment that is rich in multipath, which has traditionally been a challenge for wireless communications [4].

There has also been interest in extending MIMO communication concept to ad hoc network applications. For this, the medium access protocols such as IEEE 802.11n are introduced. As a consequence, WLANs that use IEEE 802.11n standard enable much higher spectral efficiencies than that of the traditional wireless systems providing high data throughput, improved system performance, and so on [3], [5]. Furthermore, to maximize the propagation channel capacity for IEEE 802.11n systems full channel state information (CSIT) can be made available at the transmitter [6].

MIMO ad hoc network that contains multi-antenna nodes helps to establish the access to all active users by allowing simultaneous transmissions in a way that leaves no idle channels. Such facilities are rather difficult to deploy in an ad hoc network with omnidirectional antennas nodes due to the lack of a central node. In addition, by simultaneous transmissions, MIMO system can also exploit the multiuser diversity, propagation channel diversity potentially improving the overall performance of ad hoc network.

In this paper we analyze the performance in terms of signal-to-interference-plus-noise-ratio (SINR) for two MIMO physical layers [7], [8]: 1) Spatial multiplexing; this scheme, allows the transmission of multiple independent data streams on a set of transmit antennas using CSIT. We consider the case of dominant eigenmode transmission when the transmitted energy has put into the larger eigenvalue and eigenvector of the channel matrix. 2) spatial multiplexing with adaptive weights; this scheme, as well as the first one, uses CSIT to put the energy into the larger eigenvalue and eigenvector of the channel matrix, but the beamforming weights are adjusted to the interferer scenario.

II. MIMO Ad Hoc Wireless Channel and Signal Models

Let us consider an ad hoc network with simultaneously communicating transmitter-receiver node pairs. Each transmitter is equipped with M_t transmit antennas and receiver with M_r receive antennas as Fig. 1 shows. All MIMO ad hoc nodes communicate in the Rayleigh fading propagation channel with a rich scattering environment, and each transceiver-receiver pair attempts to suppress the interference through the use of multiple receiver antennas. We assume also that all nodes have identical power constraint.

We introduce the channel matrix model for the arbitrary transmitter-receiver pair as

$$\mathbf{H} = \mathbf{R_r}^{1/2}\mathbf{H}_{\mathcal{N}}\mathbf{R}_t^{1/2}, \tag{1}$$

where $\mathbf{H}_{\mathcal{N}} \in \mathbb{C}^{M_r \times M_t}$ is the i.i.d. complex values $\mathcal{CN}(0,1)$, which are the collection of all channel propagation coefficients, $\mathbf{R}_r$ and $\mathbf{R}_t$ are receiver and transmitter correlation matrix, respectively. Assuming that $\mathbf{R}_r$ is the identity matrix,

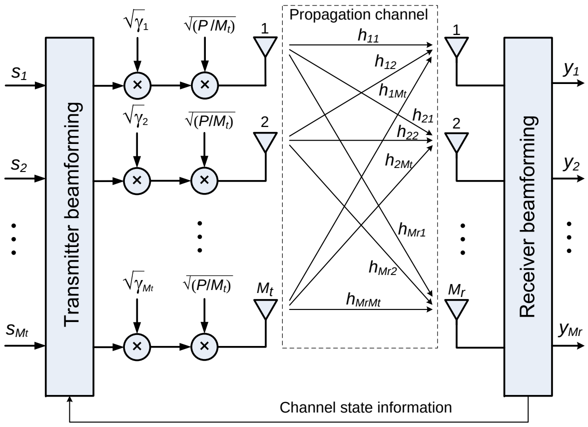

Fig. 1. Transmitter-receiver node pair in MIMO ad hoc network

we consider the case with correlation at the transmitter side only. For the uniform linear array, the correlation coefficient r_{ij} between ith and jth transmitting antennas is [3]

$$r_{ij} = J_0\left[2\pi(i-j)d/\lambda\right], \tag{2}$$

where $J_0(x)$ is the zero order Bessel function of the first kind, and d/λ is an inter-element distance to the carrier wavelength ratio.

Furthermore, we ignore the large scale propagation attenuation of the received signal, assuming that $\sum_{j=1}^{M_t} E\{|h_{ij}|^2\} = M_t, i = 1, 2, \ldots, M_r$, where $E\{\cdot\}$ is the expectation operator, and h_{ij} are the elements of the matrix $\mathbf{H}$ [9]. This implies that each of the receiver antenna receives a power, which is equal to the total transmitted power P.

We assume that channel matrix can be estimated at the receiver side, and the resulting CSIT is retransmitted to the transmitter through a feedback channel. As a result, the singular value decomposition $\mathbf{H} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^{\dagger}$ could be available, where $\mathbf{U}$ and $\mathbf{V}$ are the matrices with orthonormal properties, $\mathbf{\Sigma}$ is a singular values matrix, and $\dagger$ is a conjugate and transpose symbol.

Since the transmitted signal $\mathbf{s} \in \mathbb{C}^{M_t \times 1}$ is column vector with i.i.d. standard Gaussian entries $s_i,\ i = 1, 2, \ldots, M_t$, the covariance matrix of the transmitted signal is

$$\mathbf{R}_{ss} = E\left\{\mathbf{s}\mathbf{s}^{\dagger}\right\} = \mathbf{\Upsilon}\mathbf{I}_{M_t}, \tag{3}$$

where $\mathbf{\Upsilon} = diag\left\{\gamma_1, \gamma_2, \ldots, \gamma_{M_t}\right\}$, are subchannel power allocation factors, which is computed, as a rule, with the water filling algorithm [10], helping to feed each transmitting antenna with corresponding gain to achieve the maximum data throughput, and $\mathbf{I}_{M_t}$ is the identity matrix of the corresponding size.

Using beamforming approach the receiver node input-output signals relationship at the *i*th virtual MIMO channel is given by

$$y_i = \sqrt{P}\mathbf{w}_r^{\dagger}\mathbf{H}\mathbf{w}_t\gamma_i s_i + \mathbf{w}_r^{\dagger}\mathbf{n}, \tag{4}$$

where $\mathbf{w}_t \in \mathbb{C}^{M_t \times 1}$ and $\mathbf{w}_r \in \mathbb{C}^{M_r \times 1}$ are transmitter and receiver beamformer column vectors, respectively, with the unit length, e.i., $\|\mathbf{w}_r\|_2^2 = \|\mathbf{w}_t\|_2^2 = 1$, $\|\cdot\|_2$ denotes the 2-norm, and $\mathbf{n} \in \mathbb{C}^{M_r \times 1}$ with the entries $\mathcal{CN}(0, 1)$ and with the power N_0.

The set of transmitter and receiver beamformer vectors are chosen to optimize some cost function, for example, signal-to-noise-ratio (SNR), for each channel realization. The SNR in output of *i*th virtual subchannel of a receive node yields

$$SNR_i = \frac{P\gamma_i|\mathbf{w}_r^{\dagger}\mathbf{H}\mathbf{w}_t|^2}{\|\mathbf{w}_r\|_2^2 N_0}. \tag{5}$$

Throughout the paper we use another cost function, that is, signal-to-interference-plus-noise-ratio (SINR), because in real ad hoc communication networks with multiple antennas in use more then one node can be active, and as a consequence interfering each other.

III. Spatial multiplexing with SVD of the channel matrix

Let us consider the communication scenario with the node of interest and *K* other nodes that can operate in both receiving and transmitting mode. The node of interest can establish the transceiver link with any other node. Hence, the receiving node receives the signal of interest $\mathbf{s}^{(d)}$ from a desired node , as well as K interferer signals $\mathbf{s}^{(k)},\ k = 1, 2, \ldots, K$, which transmitted by interferer nodes.

The set of the channel matrixes relative to the desire node is $\{\mathbf{H}^{(d)}, \mathbf{H}^{(1)}, \mathbf{H}^{(2)}, \ldots, \mathbf{H}^{(k)}, \ldots, \mathbf{H}^{(K)}\}$. Hence, the CSIT related to all $K+1$ nodes could be available at this node.

Let the SVD of the matrix $\mathbf{H}^{(k)}$ is

$$\mathbf{H}^{(k)} = \mathbf{U}^{(k)}\mathbf{\Sigma}^{(k)}\mathbf{V}^{(k)\dagger}, \tag{6}$$

where $\mathbf{U}^{(k)} \in \mathbb{C}^{M_r \times M_r}$ and $\mathbf{V}^{(k)} \in \mathbb{C}^{M_t \times M_t}$ are corresponding receiver and transmitter beamforming matrices with the orthonormal properties, $k = d, 1, 2, \ldots, K$, $\mathbf{\Sigma}^{(k)}$ is a matrix with the singular value σ_i through the main diagonal, $i = 1, 2, \ldots, r$, and r is the rank of the matrix $\mathbf{H}^{(k)}$. The desire received node output vector in the presence of K interferer transmitter is

$$\begin{aligned}\mathbf{y} =& \sqrt{P^{(d)}}\mathbf{U}^{(d)\dagger}\mathbf{H}^{(d)}\mathbf{V}^{(d)}\mathbf{\Upsilon}^{(d)\frac{1}{2}}\mathbf{s}^{(d)} \\ &+ \sum_{k=1}^{K}\sqrt{P^{(k)}}\mathbf{U}^{(d)\dagger}\mathbf{H}^{(k)}\mathbf{V}^{(k)}\mathbf{\Upsilon}^{(k)\frac{1}{2}}\mathbf{s}^{(k)\dagger} + \mathbf{U}^{(d)\dagger}\mathbf{n}.\end{aligned} \tag{7}$$

We rewrite (7) with (6) as

$$\begin{aligned}\mathbf{y} =& \sqrt{P^{(d)}}\mathbf{\Sigma}^{(d)}\mathbf{\Upsilon}^{(d)\frac{1}{2}}\mathbf{s}^{(d)} \\ &+ \sum_{k=1}^{K}\sqrt{P^{(k)}}\mathbf{\Sigma}^{(k)}\mathbf{U}^{(d)\dagger}\mathbf{U}^{(k)}\mathbf{\Upsilon}^{(k)\frac{1}{2}}\mathbf{s}^{(k)} + \mathbf{U}^{(d)\dagger}\mathbf{n}.\end{aligned} \tag{8}$$

Then, the resulting SINR in the output of the virtual subchannel in the dominant eigenvalue mode is

$$SINR = \frac{P^{(d)}\gamma_1^{(d)}\sigma_1^{(d)2}}{\sum_{k=1}^{K} P^{(k)}\gamma_1^{(k)}|\sigma_1^{(k)2}\mathbf{u}_1^{(d)\dagger}\mathbf{u}_1^{(k)}|^2 + N_0}. \tag{9}$$

where $\mathbf{u}_1^{(k)}$ and $\sigma_1^{(k)2}$ are dominant eigenvector and eigenvalue of the matrix $\mathbf{H}^{(k)}$.

As (9) shows, in the reach propagation environment it is possible that $\mathbf{u}_1^{(d)\dagger}\mathbf{u}_1^{(k)} \rightarrow 0$. However, in the general case $\mathbf{u}_1^{(d)\dagger}\mathbf{u}_1^{(k)} \neq 0$. It is due to the Angle of Arrival (AoA) of interfere signals are randomly distributed, and the suppression of all interference in the output of desire node is a challenging problem.

IV. Spatial multiplexing with adaptive beamforming

Depending on either the signal should be received with a certain gain (desirable signal), or should be suppressed as much as possible (interferer signal), the strategy of choosing the transmit ($\mathbf{w}_t$) and receive ($\mathbf{w}_r$) weights in the adaptive beamforming mode are different.

We consider three cases of relationships between transmitter and receiver nodes, which includes either a desired communication pair link that requires to maximize SINR on the receiving node, or undesired potentially interfere link that requires to minimize interference-to- noise-ratio (INR) on the receiving node [8].

1) If in the receiver node weight vector $\mathbf{w}_r^{(d)}$ is adjusted to desired signal $\mathbf{s}^{(d)}$, which is transmitted by desirable transmitted node then we can choose $\mathbf{w}_t^{(d)}$ to satisfy $\mathbf{w}_r^{(d)\dagger}\mathbf{H}^{(d)}\mathbf{w}_t^{(d)} = 1$.
2) If in the receiver node weight vector $\mathbf{w}_r^{(d)}$ is adjusted to desired signal $\mathbf{s}^{(d)}$, which is transmitted by some desirable transmitted node, and in the same time kth transmitted node tries to transmit undesirable signal (interferer) to that receiver node, then we chose $\mathbf{w}_t^{(k)}$ such that the transmitter do not create interference at the receiver, or $\mathbf{w}_r^{(d)\dagger}\mathbf{H}^{(k)}\mathbf{w}_t^{(k)} = 0$.
3) If the transmitter node is communicating using a fixed $\mathbf{w}_t^{(d)}$, and the receiver nodes with $\mathbf{w}_r^{(k)}$ wish to suppress the interference from this transmitter, then $\mathbf{w}_r^{(k)}$ should satisfy $\mathbf{w}_r^{(k)\dagger}\mathbf{H}^{(k)}\mathbf{w}_t^{(d)} = 0$. To obtain $\mathbf{w}_r^{(k)}$ the knowledge of the nearest active transmit nodes is needed.

We consider the dominant eigenvalue transmission mode or beamforming mode that takes into account only larger eigenvalue and corresponding to it eigenvector of the channel matrix. Then, the desire receive node output signal in the presence of K interferer transmitter is

$$y = \sqrt{P^{(d)}}s^{(d)}\mathbf{w}_r^{(d)\dagger}\mathbf{H}^{(d)}\mathbf{w}_t^{(d)} + \sum_{k=1}^{K}\sqrt{P^{(k)}}s^{(k)}\mathbf{w}_r^{(d)\dagger}\mathbf{H}^{(k)}\mathbf{w}_t^{(k)} + N_0. \tag{10}$$

The first term in (10) is the voltage of the desire signal, while the second one is a collection of all interferer signal in the output of the desire node, which should be suppressed as deeply as possible. The resulting SINR yields

$$\mathit{SINR} = \frac{P^{(d)}\gamma^{(d)}|\mathbf{w}_r^{(d)\dagger}\mathbf{H}^{(d)}\mathbf{w}_t^{(d)}|^2}{\sum_{k=1}^{K}P^{(k)}\gamma^{(k)}|\mathbf{w}_r^{(d)\dagger}\mathbf{H}^{(k)}\mathbf{w}_t^{(k)}|^2 + N_0}. \tag{11}$$

As follows from (11), satisfying all three cases of relationships between transmitter and receiver nodes presented above substantially larger SINR can be achieved than previous discussed approach.

To satisfy the case 1 and case 2 we suppose that $\mathbf{w}_t^{(k)}$ already fixed in the transmitted nodes, and to find $\mathbf{w}_r^{(d)}$ we need to solve the system of linear equations

$$\begin{aligned} \mathbf{w}_r^{(d)\dagger}\mathbf{H}^{(d)}\mathbf{w}_t^{(d)} &= 1, \\ \mathbf{w}_r^{(d)\dagger}\mathbf{H}^{(1)}\mathbf{w}_t^{(1)} &= 0, \\ &\dots \\ \mathbf{w}_r^{(d)\dagger}\mathbf{H}^{(K)}\mathbf{w}_t^{(K)} &= 0, \end{aligned} \tag{12}$$

In the matrix form (12) can be rewritten as

$$\mathbf{w}_r^{(d)\dagger}\mathcal{H}_k = \mathbf{q}^T, \tag{13}$$

where $\mathcal{H}_k \in \mathbb{C}^{M_r \times K}$ consists the columns $\mathbf{H}^{(k)}\mathbf{w}_t^{(k)}$, $k = d, 1, 2, \dots, K$, and $\mathbf{q} = [1, 0, \dots, 0]^T$. The solution of (13) is

$$\mathbf{w}_r^{(d)} = \mathcal{H}_k^{\sharp\dagger}\mathbf{q}, \tag{14}$$

where $\sharp$ is a pseudo inverse matrix operation sign.

To satisfy the case 3 we suppose that $\mathbf{w}_r^{(k)}$ already fixed in the nearest receiving nodes, and to find $\mathbf{w}_t^{(d)}$ we need to solve another system of linear equations

$$\begin{aligned} \mathbf{w}_r^{(d)\dagger}\mathbf{H}^{(d)}\mathbf{w}_t^{(d)} &= 1, \\ \mathbf{w}_r^{(1)\dagger}\mathbf{H}^{(1)}\mathbf{w}_t^{(d)} &= 0, \\ &\dots \\ \mathbf{w}_r^{(K)\dagger}\mathbf{H}^{(K)}\mathbf{w}_t^{(d)} &= 0. \end{aligned} \tag{15}$$

Similarly, (15) in the matrix form is

$$\mathcal{Q}_k\mathbf{w}_t^{(d)} = \mathbf{q}, \tag{16}$$

where $\mathcal{Q}_k \in \mathbb{C}^{K \times M_t}$ is a matrix with the rows $\mathbf{w}_r^{(k)\dagger}\mathbf{H}^{(k)}$, $k = d, 1, 2, \dots, K$. The corresponding solution of (16) is

$$\mathbf{w}_t^{(d)} = \mathcal{Q}_k^{\sharp}\mathbf{q}, \tag{17}$$

Afterward, the resulting vectors should be normalized as $\mathbf{w}_r^{(d)}\left(\mathbf{w}_r^{(d)\dagger}\mathbf{w}_r^{(d)}\right)^{-1/2}$, and $\mathbf{w}_t^{(d)}\left(\mathbf{w}_t^{(d)\dagger}\mathbf{w}_t^{(d)}\right)^{-1/2}$.

V. Simulation Result

We use the numerical example to examine the output SINR of a MIMO ad hoc network receiving node. The uniform linear array (ULA) with $\lambda/2$ space element distance with $M_t = M_r = 6$ is assumed. The desire receive node operates in the presents of one desire transmit node as well as two neighbor interferer nodes, as Fig. 2 shows. The propagation matrixes $\{\mathbf{H}^{(d)}, \mathbf{H}^{(1)}, \mathbf{H}^{(2)}\}$ has been generated using (1) and (2). The input random signal with 16-QAM symbol mapping is used. Two transmitting schemes are considered:

1) spatial multiplexing with SVD of the channel matrix and the dominant eigenmode transmission (SM-DET).

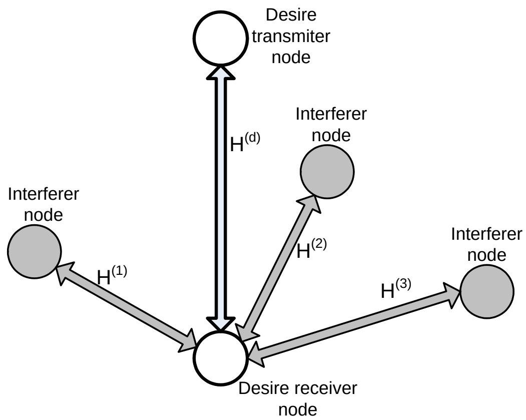

Fig. 2. MIMO ad hoc network simulation scenario.

2) spatial multiplexing with adaptive weights (SM-AW), where the beamforming weights are adjusted to the interferer scenario.

The gain $\frac{(SINR)_{SM-AW}}{(SINR)_{SM-DET}}$ vs. INR is presented in Fig.2, where the curve 1 is the case when angle of arrival (AoA) of the signals from interferer nodes 1 , 2, and 3 are -30.0, 30.0, and 56.4 degrees, respectively. In this case the orthogonal conditions are not committed, i.e., $\mathbf{u}_1^{(d)\dagger}\mathbf{u}_1^{(k)} \neq 0,\ k = 1, 2, 3$, and the gain of the SM-AW over SM-DET are significant, linearly increasing with INR.

The curve 2 is the case when AoA of signals from interferer nodes 1, 2, and 3 are -19.5, 19.5, and 56.4 degrees, respectively. For this case, $\mathbf{u}_1^{(d)\dagger}\mathbf{u}_1^{(k)} = 0,\ k = 1, 2$, and $\mathbf{u}_1^{(d)\dagger}\mathbf{u}_1^{(3)} \neq 0$, i.e, the nodes 1 and 2 maintain the orthogonal transmission, but node 3 do not. The gain yet significant, but not so much.

The curve 3 is the case when AoA of signals from interferer nodes 1, 2, and 3 are -19.5, 19.5, and 41.8 degrees, respectively. In this case, $\mathbf{u}_1^{(d)\dagger}\mathbf{u}_1^{(k)} = 0,\ k = 1, 2, 3$, and no difference between two transmission schemes.

As Fig. 3 shows, for the typical communication scenario with INR about 15 dB SM-AW has a SINR advantage of approximately 28 dB over SM-DET. As a drawback of SM-AW can be mentioned the necessity of additional intensive computation with the complexity $O(N_t^3)$ to weight vector adjustment.

VI. Conclusion

We consider the performance of two MIMO schemes that can be used in wireless ad hoc applications: 1) spatial multiplexing with SVD of the channel matrix and the dominant eigenmode transmission (SM-DET), 2) spatial multiplexing with adaptive weights (SM-AW), where the beamforming weights are adjusted to the interferer scenario. As the simulation result shows, SM-AW has a significant SINR gain over SM-DET, when the orthogonal conditions between interferer and receive nodes are not held. In the same time, when the orthogonal conditions are complied, both schemes have the similar performance.

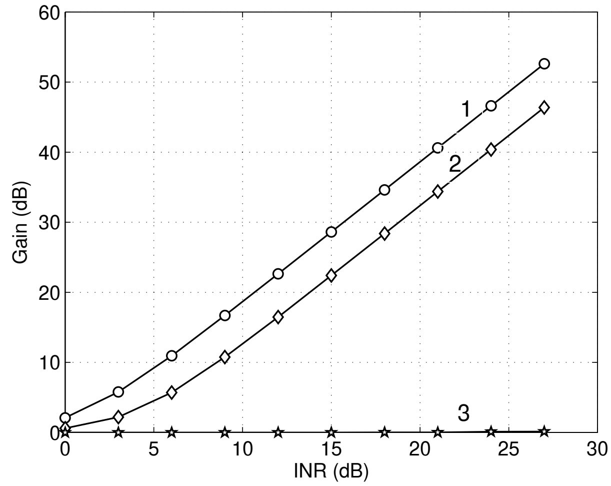

Fig. 3. MIMO ad hoc node performance.

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